



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 28

Yield Shifts & Immunization

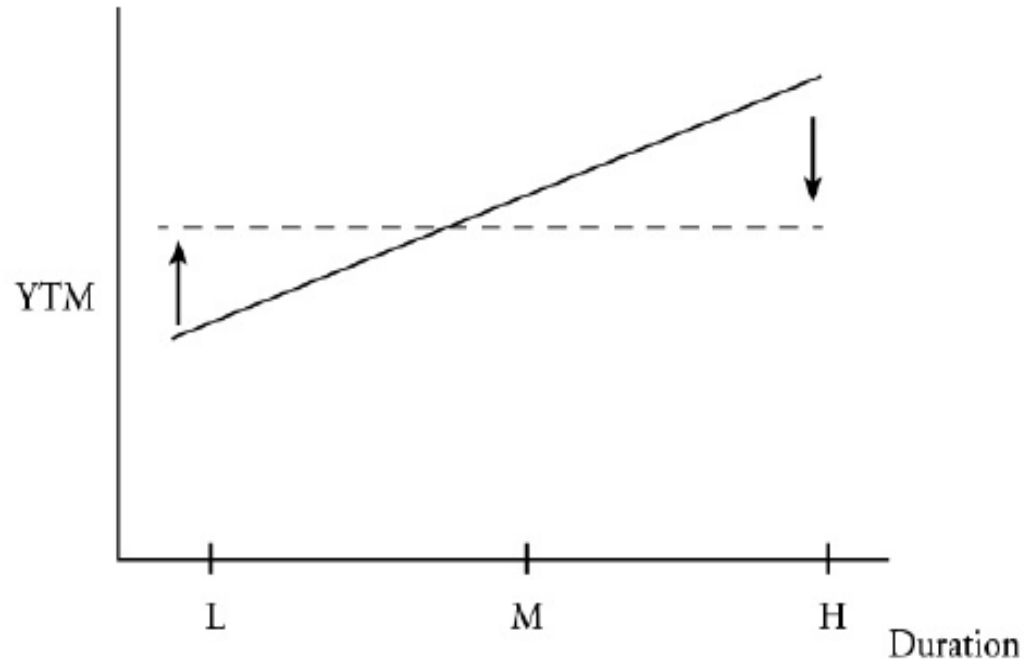
J P Singh

Department of Management Studies



YIELD CURVE FLATTENING

- Assume that rates do not change for the M duration liability.
- Yield L increases while yield H decrease relative to yield M.

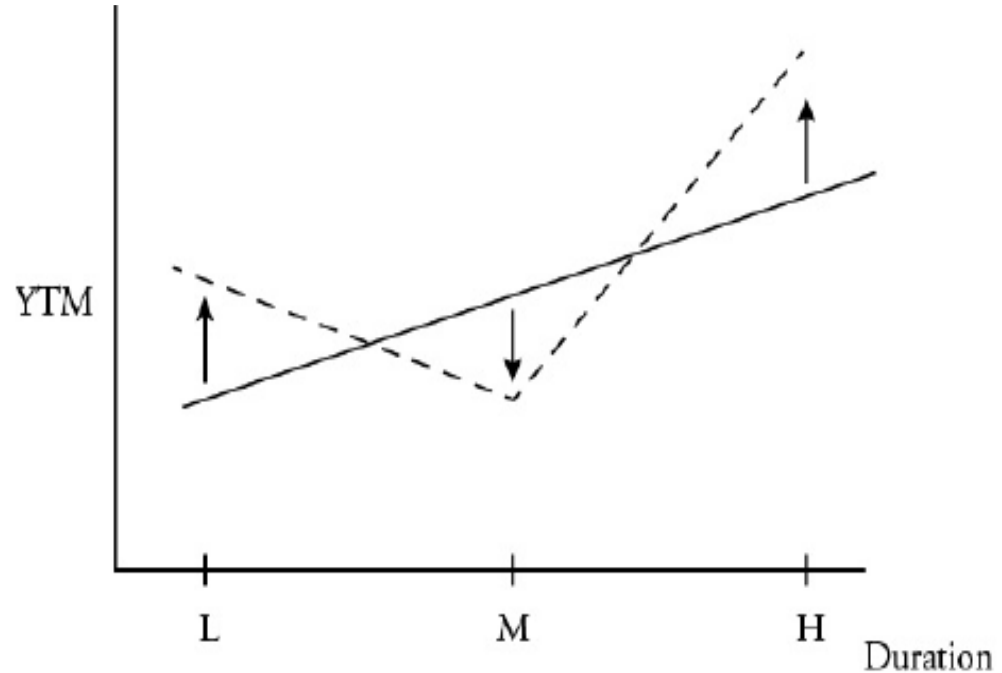


- The portfolio market value will increase because the increase in value of the longer duration bond will exceed the decrease in the value of the shorter duration bond.
- PVL will be unchanged with no change in yield M .
- PVA is now above PVL.

- That, by itself, does not indicate the strategy will succeed.
- If portfolio IRR decreases sufficiently, the required FV of assets to meet the payout may not be reached.
- Recall that portfolio IRR would tend to decrease below a single point M YTM with a flatter curve).
- This indicates that a flattening curve may create structural risk.

POSITIVE BUTTERFLY TWIST

Yield L and H increase. Yield M decreases.



- The portfolio market value will decrease as both yield L and H increase.
- PVL will increase as yield M decreases.
- PVA is now below PVL.
- That is certainly detrimental, but it is possible the strategy could succeed if the portfolio IRR increases enough versus the decrease in liability discount rate.
- This indicates that the positive butterfly may create significant structural risk.

EXAMPLE

- Consider the barbell strategy consisting of longs in two bonds L & H with equal money weights (INR 1,000) of 1 year and 9 year ZCBs and short in a ZCB liability M of INR 4,022.71 at $t=5$ years. All the bonds are trading at a YTM of 15% p.a. Assume that there is a instantaneous non-parallel shift of the yield curve due to which we have $S_{01} = 17\%$, $S_{05} = 12\%$ & $S_{09} = 18\%$. Evaluate the performance of the strategy.

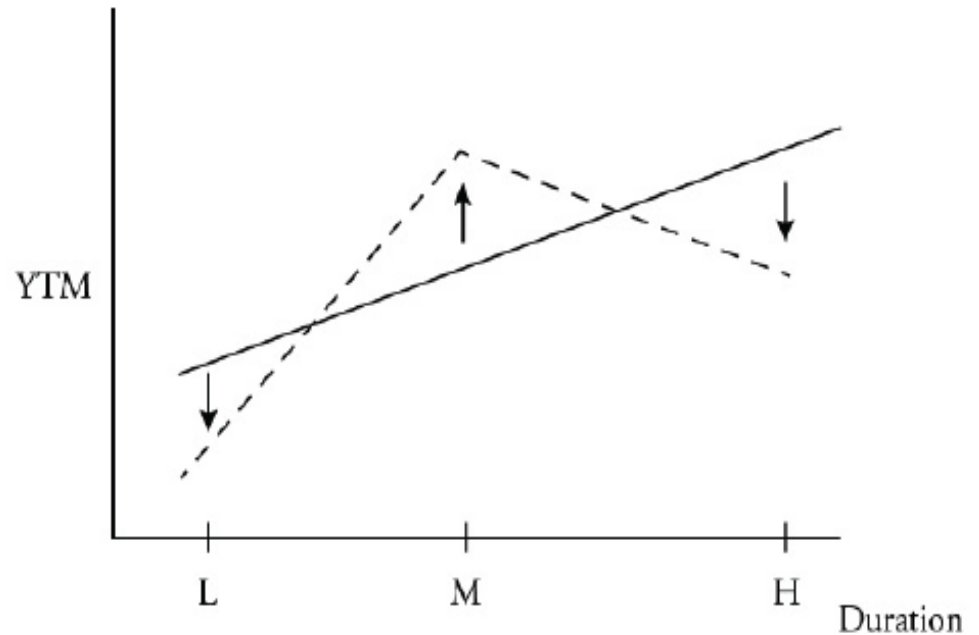
SOLUTION

- $P_L = \frac{1,000 \times 1.15}{1.17} = \frac{1,150}{1.17} = 982.91$
- $P_H = \frac{1,000 \times 1.15^9}{1.18^9} = \frac{3,517.88}{1.15^9} = 793.12$
- $P_{BARBELL} = P_L + P_H = 1,776.04$
- $P_M = \frac{4022.71}{1.12^5} = 2282.59$
- $P_{BARBELL} < P_M$

- The new IRR of the strategy can be calculated by solving: $1,776.04 = \frac{1,150}{(1+y)} + \frac{3,517.88}{(1+y)^9}$. We get $y=17.88\%$.
- Hence, value of barbell & liability at $t=5$ years:
- Value of liability = $2282.59 \times 1.12^5 = 4022.71$
- Value of barbell = $1776.04 \times 1.1788^5 = 4042.51$

NEGATIVE BUTTERFLY TWIST

Yield L and H decrease. Yield M increases.



- The portfolio market value will increase as both yield L and H decrease.
- PVL will decrease as yield M increases.
- PVA is now clearly above PVL.
- That is certainly favorable, but does not guarantee the strategy will succeed if the portfolio IRR decreases too much in relation to the increase in liability discount rate.
- It again indicates the possibility of significant structural risk.

RULES FOR LIABILITY IMMUNIZATION

- The risk in immunization is higher when the change in portfolio IRR does not match the change in yield of the replicating zero or is insufficient to fund the liability at the new level of rates.
- This structural risk can be reduced by reducing the dispersion of asset cash flows around the liability cash flow.

- This is not surprising because if you make dispersion 0, you have a zero-coupon bond and a perfect cash flow match to the single liability.
- Now, recall the earlier equation for determining convexity from duration and dispersion; reducing dispersion is directly related to reducing convexity

RULES FOR DURATION MATCHING

- Initial portfolio market value (PVA) equals (or exceeds) PVL depending on whether the initial portfolio IRR equals (or is less than) the initial discount rate of the liability.
- Portfolio Macaulay duration matches the due date of the liability ($D = D_L$).
- Minimize portfolio convexity (to minimize dispersion of asset cash flows around the liability and reduce risk to curve reshaping).

- Regularly rebalance the portfolio to maintain the duration match as time and yields change. (But also consider the tradeoff between higher transaction costs from more frequent rebalancing versus the risk of allowing durations to drift apart.)
- It may seem strange to require minimizing convexity of assets when $+C$ is good for immediate price change. **But that ignores the real issue of immunization, failing to reach the FV needed to pay the liability.**

SUMMARY

- Immunization can be interpreted as zero replication, meaning a successful immunization will replicate the price and yield path of a zero-coupon bond that could have been used for a perfect cash flow match immunization.
- Immunization can be used to fund liabilities with a high degree of certainty. The assets are dedicated to this purpose and all cash flows are reinvested until needed for payout.

CASH FLOW BASED IMMUNIZATION

- **Cash flow matching is without risk, assuming there are no defaults.**
- **Bonds are bought and held in sufficient amount and pay date to meet the liabilities. It is the most restrictive strategy, and so typically costs more (has lowest return).**



DURATION BASED IMMUNIZATION

- Duration matching achieves similar results, but is less restrictive in the assets selected.
- Matching Macaulay duration of the assets to liabilities balances the exposure between price and reinvestment risk.
- Duration and other portfolio statistics should be based on portfolio yield (IRR).

- To immunize a single-period liability:
- Initial PVA equals (or exceeds) PVL.
- Match Macaulay durations ($D_A = D_L$).
- Minimize portfolio convexity but keep it greater than liability convexity.
- Rebalance the portfolio to maintain the duration match.

- Immunization (duration matching) issues include the following:
- The assets have greater convexity than the single date liability; therefore, the portfolio benefits from large parallel shifts but is at risk from curve twists (nonparallel shifts).
- Minimizing convexity minimizes this structural risk.

WEIGHTED AVERAGE VS ACTUAL PORTFOLIO STATISTICS



PORTFOLIO STATISTICS

- Portfolio yield (meaning YTM), duration, dispersion of cash flows, and convexity are commonly computed as weighted averages based on market value weighting of each holding in the portfolio.
- These average computations are less accurate than portfolio statistics computed directly from the portfolio's aggregate cash flows.



- **Portfolio statistics obtained directly from portfolio cash flow estimates should be used for ALM work rather than traditional weighted average calculations based on each bond.**
- **The difference in the two approaches is determined by the shape of the yield curve:**
- **With a flat yield curve, there is no difference.**
- **In an upward-sloping yield curve, portfolio duration and IRR (YTM) will be higher-than weighted average duration and YTM of the bonds because portfolio statistics reflect all cash flows (and return) to be received and the longer maturity bonds will impact the portfolio for a longer time.**

- To illustrate this issue:
- The ytm of a portfolio is not equal to the weighted average ytm of its constituents.
- A portfolio is made of equal proportions (by value) of two securities A and B. The current price of A is 100 and its respective cashflows over the next three years is 15, 15 and 115. The price of B is 100 and the cash flows are 6, 106 and 0. Calculate the YTM of A, B and the portfolio as well.



PORTFOLIO	YTM	0	1	2	3
A	0.15	-100	15	15	115
B	0.06	-100	6	106	0
PORTFOLIO	0.112891	-200	21	121	115

